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Quenching of the Hall effect by the phase-driven current in quasi-1D systems

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Abstract. Quenching of the Hall effect in quasi-1D samples has been shown to be caused by a new quantum mechanical phenomenon in the low-magnetic-field regime wherein a current has been found to counter the current driven by the Lorentz force. This current is driven by the phase slip created between the two edges by the Hall voltage developed by the magnetic field. It has a very low frequency. The critical value of the magnetic field where the Hall effect appears has been estimated and found to be in good agreement with the experimental value. It is suggested that the quasi-1D devices may prove useful as high-speed switches.

In a seemingly unending series of interesting new phenomena unravelled by experimental and theoretical work on the Hall effect in different regimes of interest, 'quenching' of the Hall effect in quasi-one-dimensional (1D) systems at very low magnetic fields has also been recently reported [1]. We explain this new result here. It is shown to be due to an intrinsic Josephson-type effect in narrow Hall devices recently predicted by the present author [2]; the result in question is a manifestation of the *low-frequency* limit of this effect. Thus the quenching of the Hall effect provides an experimental confirmation of the predicted AC effect although only in the *low-frequency* limit.

First, we should understand the physical implications of the quenching phenomenon which may give us a clue to understanding the result. Recall the basic equation for the resistivity tensor pertaining to the Hall effect in 2D. It can be written as a pair of simultaneous equations

$$J_y \rho_{xy} = E_x - \rho_{xx} J_x \tag{1a}$$

and

$$J_x \rho_{yx} = E_y - \rho_{yy} J_y \tag{1b}$$

where an applied J_x in the $(+)x$ direction establishes E_x and a magnetic field B_z gives rise to a J_y in the $(+)y$ direction which is soon stopped by the establishment of the Hall field E_y pointing in the $(-)y$ direction; ρ_{ij} are the elements of the resistivity tensor. The quenching of the Hall effect means that $\rho_{xy} = -\rho_{yx} = 0$ which can happen if

$$E_x = \rho_{xx} J_{xx} \tag{2a}$$

and

$$E_y = \rho_{yy} J_y \tag{2b}$$

simultaneously. The condition (2a) implies that the current density J_x, *parallel to* E_x, is

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driven entirely by E_x in spite of the presence of B_z, and we see from (2*b*) that such a situation can be created only if a J_y flows *parallel* to E_y, i.e. in the $(-)y$ direction, such that the condition (2*b*) is fulfilled. That is, 'something' should start a current equal in magnitude but opposite in direction to the current J_y generated by the B_z, so that it can exactly cancel the influence of the latter. This effect will stop the Hall voltage from developing between the edges of the sample and will thus check the Hall resistance from rising above zero. So we have to understand what can give rise to a current equal and opposite to the Hall-J_y. We suggest that this current can be caused by the phase difference between the opposite edges of the sample. It is supposed to be present in the narrow Hall devices all the time in the Hall set-up [2] but, in the present situation , it plays a crucial role owing to its *very low* frequency, so low so that the wavelength of the current is larger than the width of the sample.

To start, we briefly recapitulate the results of [2]. It was shown that, if the edges of a narrow 2D device are at a different electric potential due to the presence of a B_z and if the width of the sample is of the order of the phase coherence length so that the states at the two edges can be taken to be coupled, then a current arises from a transition between a state in which an electron is on one side and a state in which it is on the other. Thus the relative phase of the two edges oscillates at a beat frequency given by

$$\partial\varphi/\partial t = eV_{\mathrm{H}}/\hbar \tag{3}$$

where φ is the phase difference between two points at the opposite edges (defined modulo 2π) and V_{H} is the Hall voltage. The oscillating φ drives an AC of the same frequency and is given by

$$J = J_{\mathrm{c}} \sin\varphi \tag{4}$$

where J_{c} is the critical current when $\varphi = \pi/2$. The coupling of the two edges reduces the system energy by an amount

$$E_{\mathrm{c}} = (\hbar J_{\mathrm{c}}/e)\cos\varphi . \tag{5}$$

Now, coming to the present problem, we note that when $B_z \simeq 0$ the edge currents are no longer well defined because the electron distribution will be more or less uiniform across the width of the sample. So, we cannot talk in terms of the wavefunctions localised along the edges; instead we have a very flat wavefunction as wide as the sample which is represented as

$$\psi(r) = |\psi(r)| \exp[\mathrm{i}\alpha(r)] \tag{6}$$

where $|\psi(r)|^2$ is the electron density *which remains almost constant across the width* and α is the phase explained in detail in [2]. For this slowly varying ψ, we can write in the y direction (i.e. along the width)

$$\hbar(\partial\alpha/\partial y) = (1/|\psi|^2)\langle\psi| -\mathrm{i}\hbar(\partial/\partial y)|\psi\rangle \equiv p_y = m^* v_y + (e/c)A_y \tag{7}$$

where p_y, v_y and A_y are the y components of the momentum, velocity and the vector potential (defined as $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$); m^* is the effective mass of the electron. This gives us a current in the y direction (we shall see later whether it is $+y$ or $-y$)

$$J_y = (|\psi|^2 e/m^*)[\hbar(\partial\alpha/\partial y) - (e/c)A_y]. \tag{8}$$

If the width is very small, we can write it as

$$J_y = \frac{|\psi|^2 e\hbar}{m^* w}\left(\alpha_2 - \alpha_1 - \frac{e}{\hbar c}\int_1^2 A_y\,\mathrm{d}y\right) \tag{9}$$

where the quantity inside the parentheses is the 'gauge-invariant phase difference' between the opposite edges designated as 1 and 2; α_1 and α_2 represent the phases at the two edges, and w is the width of the sample. In the limits of vanishing w and vanishing B, the integral in (9) goes to zero‡. So, for the quasi-1D samples under a vanishing magnetic field, we can write,

$$J_y = J_0 \varphi \qquad J_0 \equiv |\psi|^2 e\hbar/m^* w \qquad \varphi \equiv \alpha_2 - \alpha_1. \tag{10}$$

This is nothing, but the *small-*φ limit of equation (4) for the quasi-1D system. The φ can be small if, firstly, the phase coherence is very good and the phase is uniform across the width, so much so that we can call the system to be in a state of 'near phase rigidity' and, secondly, $V_{\mathrm{H}} \simeq 0$, because, as we see from equation (3), V_{H} can cause the phases at the edges to slip with respect to each other. Both these conditions are met in the quasi-1D samples ($w \simeq 100$ nm) when $B_z \simeq 0$ and, therefore, equation (10) pertains to the systems and conditions of our interest here. The E_{c} can be written for them as

$$E_{\mathrm{c}} \simeq |\psi|^2 h^2/8\pi^2 m^* w \qquad (\text{for } \varphi \simeq 0). \tag{11}$$

Note that $E_{\mathrm{c}} \propto w^{-1}$ and, more importantly, $E_{\mathrm{c}} \gg k_{\mathrm{B}}T$ even for T as low as 1 K (the experiments in [1] were performed at $T \simeq 4$ K). The latter is of great significance in that it allows a great deal of inter-level scattering without changing the energy of the system and so the 'near phase rigidity' is not disturbed. For $B_z \simeq 0$, the Landau quantisation is almost not there and the granularity of the energy levels due to the quasi-one dimensionality is also comparable with $k_{\mathrm{B}}T$. Both these factors can give rise to a considerable amount of inter-level scattering at $T \simeq 4$ K. In spite of this the phase rigidity remains unaffected owing to large E_{c} in the present case.

In the light of all this, we can understand the quenching of the Hall effect. Let us proceed in a systematic manner. Suppose at $t = 0$ we have $\varphi = 0$ and $B_z = 0$; then $J_y = 0$ and E_{c} is at its maximum. Now B_z is switched on. The electrons will be deflected to one of the edges, say edge 1. Consequently, the potential at edge 2 will go up and along with it the phase will also rise (see equation (3) where $\varphi = \alpha_2 - \alpha_1$). Since the phase-driven current flows from the higher phase to the lower phase [2], the phase-difference between edges 1 and 2 created by B_z will tend to drive the electrons from edge 1 to edge 2 (i.e. the current flows in the $(-)y$ directon, parallel to the E_y, the Hall field). Thus, on the one hand, B_z deflects the electrons to edge 1 and, on the other hand, the potential difference and the phase difference created as a result of this drive the electrons away from edge 1 towards edge 2. The balance between these two competing forces should decide which way the net current will eventually flow, but it is also clear that the flow of the electrons from edge 1 to edge 2 under the 'phase stress' will steadily reduce the V_{H} until it becomes zero and with it the phase-driven current will also go to zero. So the phase-driven current will never supersede the current caused by the Lorentz force and, whenever the balance is disturbed, this will lead to the development of a net V_{H}.

It is crucial, for the above effect to happen, that the phase-driven current remains 'effectively' DC in spite of the development of V_{H} as B_z is switched on. This can happen if a *quarter* of the wavelength λ of the phase-driven current is *at least* as large as the width w of the sample—the requirement imposed means that before the coupling energy E_{c} reduces to zero (which happens when $\varphi = \pi/2$) and begins to become negative (see equation (5)) the electrons driven by the phase difference should reach edge 2. As some of the electrons reach edge 2, V_{H} is reduced. Consequently the frequency $\partial\varphi/\partial t$

‡ Note that the integral in (9) goes as w/l_0^2, where l_0 is the cyclotron radius ($A_y \sim B \sim l_0^{-2}$). This approaches zero very rapidly as $B \rightarrow 0$. For a given *small* w, although $\alpha_2 - \alpha_1$ will be very small, yet for $B \simeq 0$ the integral will be negligible compared with $\alpha_2 - \alpha_1$ because of the stronger l_0^{-2} dependence in it.

decreases and λ increases; further, the coupling energy E_c increases which facilitates the flow of electrons to edge 2. In this cumulative manner, V_H approaches zero and we find that V_H cannot be maintained without accelerating the phase-driven current. This should enable us to estimate the critical value B_z^{cr} of the magnetic field up to which (starting from $B_z = 0$) the Hall effect will remain quenched. Note that equation (3) can be represented as

$$V_H \lambda = hc/e \tag{12}$$

where λ is the wavelength of the phase-driven current. Classically, V_H obeys the equation

$$v_x = V_H/wB_z \tag{13}$$

where v_x is the velocity of electrons in equilibrium in the x direction. Since v_x is maintained constant, $V_H \propto wB_z$, and, if V_H^{cr} is the classical value of V_H corresponding to B_z^{cr} up to which $\lambda \geqslant 4w$, then

$$B_z^{cr} w\lambda \simeq B_z^{cr} 4w^2 = hc/e. \tag{14}$$

This yields $B_z^{cr} \simeq 0.1$ T for $w \simeq 100$ nm which is in excellent agreement with the experimental result [1]. It was assumed here that $\varphi = 0$ at $t = 0$ which may not be strictly correct. If φ was non-zero to start with, E_c would be less than its maximum value at $t = 0$ and B_z^{cr} would be less than 0.1 T, becoming closer to the experimental estimate. One can alternatively arrive at the same estimate by equating the J_y in equation (10) to the classical J_y generated by E_x and B_z, which is simply the requirement for the quenching of the Hall effect. *Note that* $B_z^{cr} \propto w^{-2}$.

As B_z exceeds the above critical value, the condition on w and λ will break down and the current will get its AC character restored within the length scale of w which means that the accumulation of electrons on edge 1 due to B_z will remain largely undisturbed. The Hall resistance, as a result, will begin to rise above zero and will more or less follow the classical trend apart from some fluctuations about the classical value until the AC frequency is high enough to become self-averaged. At sufficiently high B_z the system will go into the quantised Hall conditions [3].

To synthesise all the results, we first recall that the quantised Hall effect [3] comes into being when, in a 2D system at low T and high B_z, the current J_x is driven *entirely* by E_y, the Hall field. This gives the result $\rho_{xx} = \sigma_{xx} = 0$ (σ_{xx} represents the diagonal conductivity). We find here that the supposedly classical regime of $B_z \simeq 0$ turns into a quantum mechanical one if at low temperatures the system is narrowed down to become *quasi-1D*, and a new phenomenon sets in—the phase-driven current [2] which remains dormant at high B_z (in that it self-averages owing to its high frequency) plays a prominent role when $B_z \simeq 0$ and stops V_H from developing under the influence of non-zero B_z. This gives a situation represented by $\rho_{xy} = \sigma_{xy} = 0$ and the j_x *being driven entirely by* E_x. (In intermediate situations when the Hall effect is neither quantised nor quenched, J_x is driven by *both* E_x and E_y.) In a 1D system, one does expect that ρ_{xy} will be zero always, i.e. there will be no Hall effect in them. However, what we find here is that even in the quasi-1D system the Hall effect can be quenched by a new underlying phenomenon and that in the limit of $w = 0$ (i.e. one-dimensionality) the Hall effect will remain quenched, in accordance with $B_z^{cr} \propto w^{-2}$, for any value of B_z. For w small but non-zero, the crossover from $\rho_{xy} = 0$ to $\rho_{xy} > 0$ at B_z^{cr} can be understood as a 'dimensionality crossover' from *effectively* 1D to 2D. For $B_z > B_z^{cr}$ the Landau level spacing (approximately $\hbar eB/m^*c$; m^* being the effective mass) becomes larger than the level spacing (approximately $\hbar^2/m^*w^2$) for the quasi-1D system of width w; in other words the magnetic length $l_0 = (\hbar/eB)^{1/2}$

becomes less than the sample width w. This recovers the proper 2D (magnetic) behaviour. It is interesting to note that the criterion $\hbar e B_z^{cr}/m^*c \simeq \hbar^2/m^*w^2$ also yields the w^{-2} dependence for B_z^{cr} as obtained earlier from equation (14); also the value of B_z^{cr} obtained from this criterion is very close to the value obtained from equation (14). This indicates that there is an intimate relationship between the quenching of the Hall effect (as deduced from the discussion up to question (14)) and the fact that the system does not behave two-dimensionally (in so far as the fact that Landau quantisation does not exceed the quasi-1D level separation is concerned).

In [4] a classical argument was proposed for the quenching of the Hall effect. Their argument, in simple terms, is that when $2l_0 > w$ the electrons are bouncing against the two edges randomly and, therefore, the Hall voltage is not developed. In a forthcoming paper, we have proved that this is not enough for the quenching phenomonon to occur. We obtain a precise (classical) condition for the quenching which is too strict to be met in an experiment.

The low-B_z phenomenon discussed here suggests that the quasi-1D devices can be useful as high-speed switches. To be more explicit, note that equation (3) can be written, for a general V (which can be applied externally as well), as

$$\varphi = \varphi_0 + (e\hbar)Vt \qquad \varphi_0 = \varphi(t=0). \tag{15}$$

When $V = 0$, the phase is uniform and 'rigid' across the width and we get $J_y = 0$ from equation (10). However, if $Vt = h/4e$ and $\varphi_0 = 0$ because the phase is uniform at $t = 0$, then $\varphi = \pi/2$ and $J_y = J_0$, i.e. the maximum current flows from edge 2 to edge 1. In other words, if between the two edges a certain V is developed (externally and *not* by the application of B_z) and is maintained for time t such that $Vt = h/4e$, then the system will draw the maximum phase-driven current of amount J_0 in the transverse direction. The current will go to zero as soon as V is lifted if, in the quasi-1D device, the electrons that have moved to edge 2 are drawn into a circuit. To estimate this, note that the above condition can be met by applying, for example, a voltage $V = 1\ \mu$V for 1 ns. The narrower is the device, the larger will be J_0 and, from the point of view of application, it is also important to note that $E_c \sim w^{-1}$ so that one can use relatively higher temperatures for narrower devices without affecting the phase rigidity. It is important that B_z should not be used to generate the voltage across the edges, for firstly it will not be able to do so until $B_z > B_z^{cr}$, and secondly above B_z^{cr} the electrons will not be able to cross over to edge 2. It would be interesting to study the quasi-1D Hall devices for the above purpose and to ascertain their merits and demerits in comparison with the Josephson devices. For instance, the quasi-1D Hall devices must be very cheap and reliable as one expects from the fact that they are now routinely fabricated.

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